

# – The Shape of Locomotion –

## A Method for Large Scale Analyses of Intra-Limb Coordination

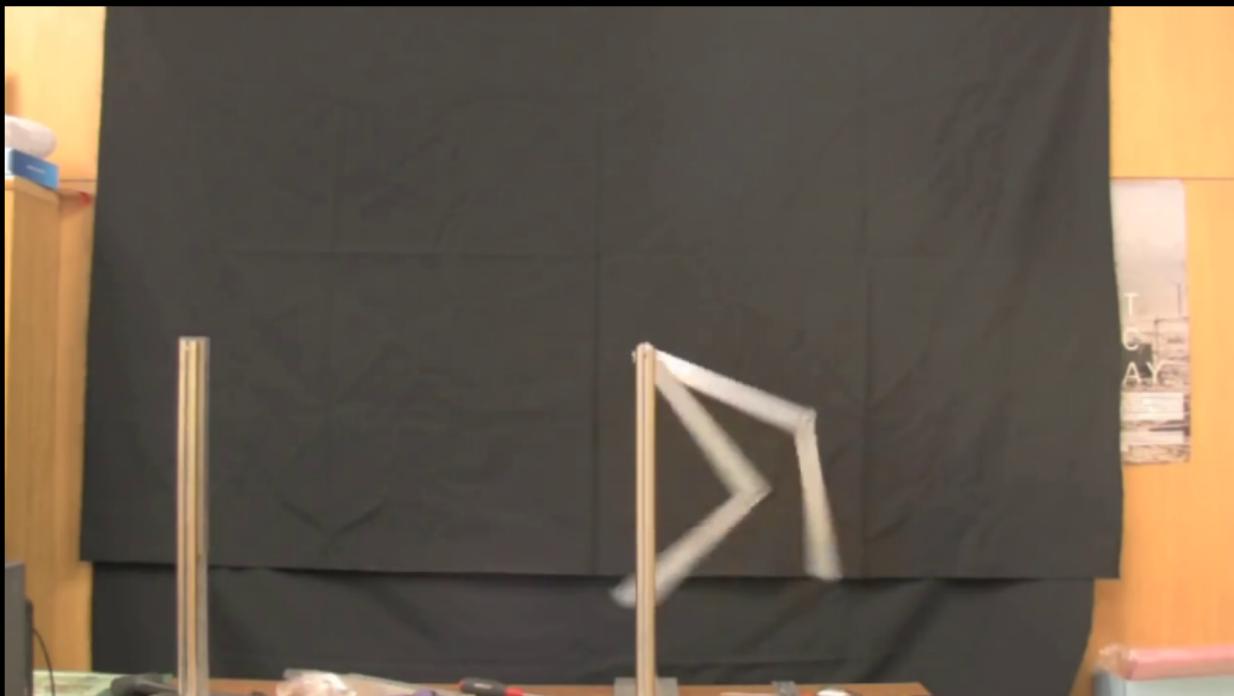
Falk Mielke\*, Chris Van Ginneken, Peter Aerts

Functional Morphology, Universiteit Antwerpen

DZG Meeting, Jena  
September 9<sup>th</sup>, 2019



# Chaos



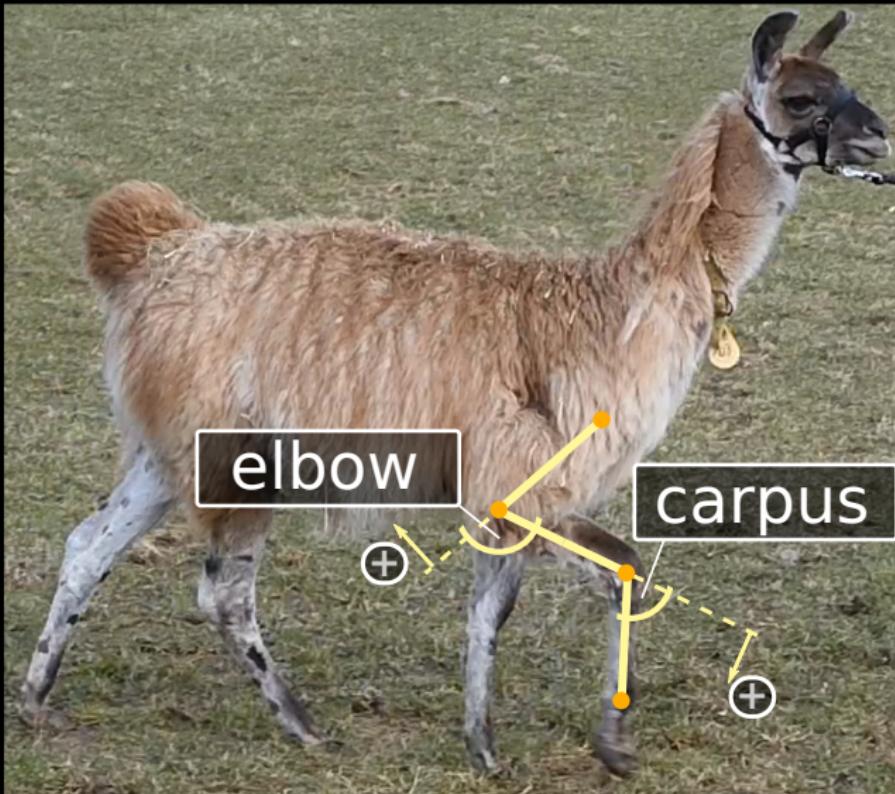
Shunichi Yamamoto, youtube N6cwXkHxLsU (Dec 2, 2011)  
"High Performance Coupled Double Pendulum", played backwards

# Control and Coordination

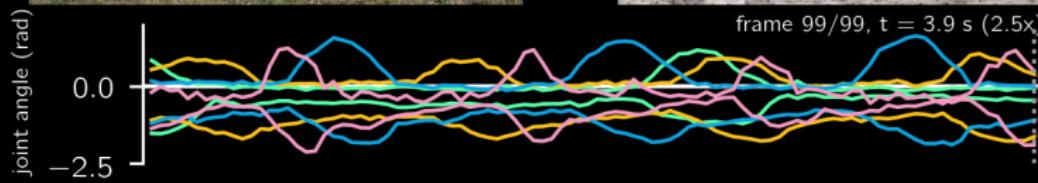


red river hog by *oschu1000*, youtube Z97wAh3IXWc (Dec 8, 2017)  
moose by *ullabri*, youtube GEZ3Ton9w6A (Apr 27, 2011)

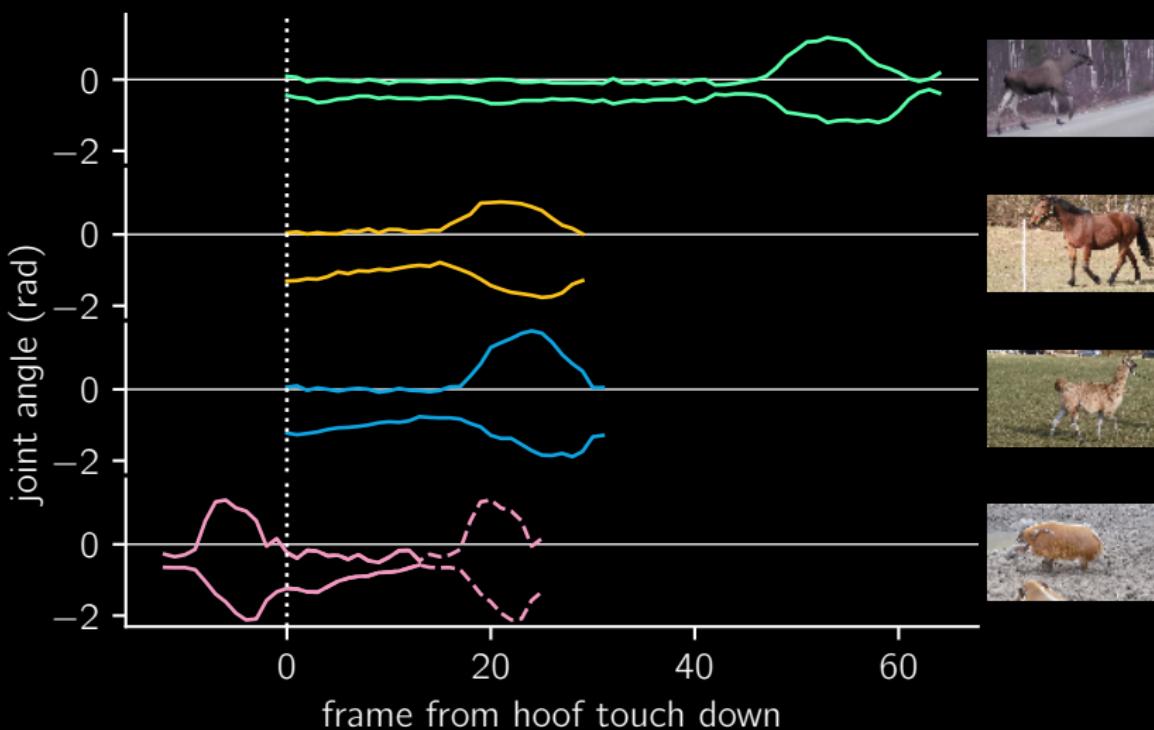
# Joint Angles



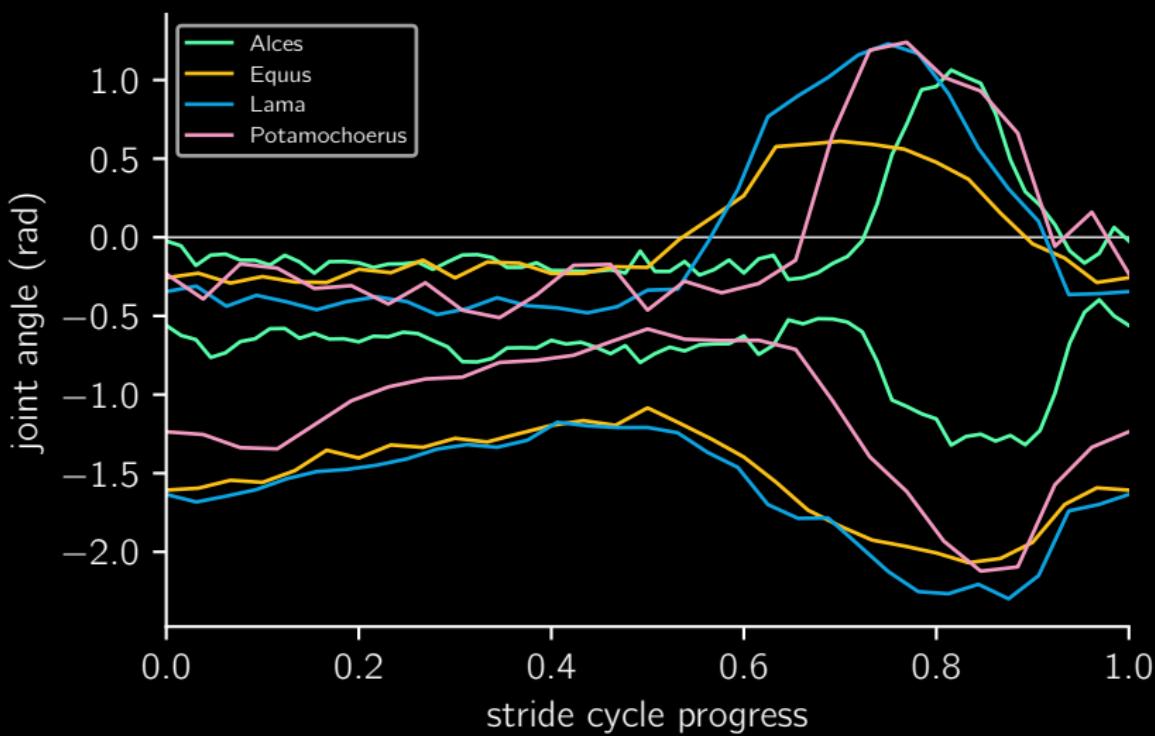
# Digitization



# Conventional Alignment



# Conventional Alignment

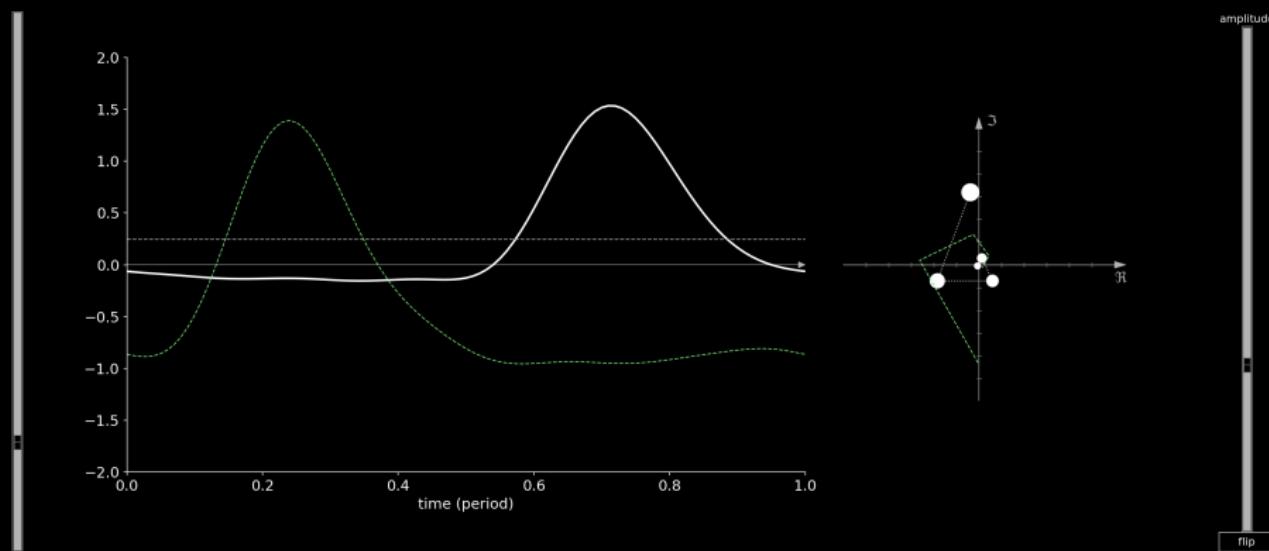


Motivation  
ooooo

Fourier Series  
●ooooo

Locomotor Shape Analysis  
○○

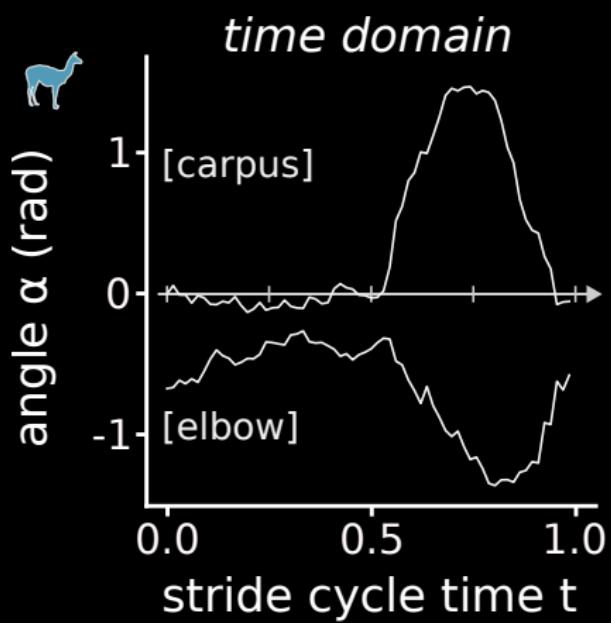
# Fourier Series



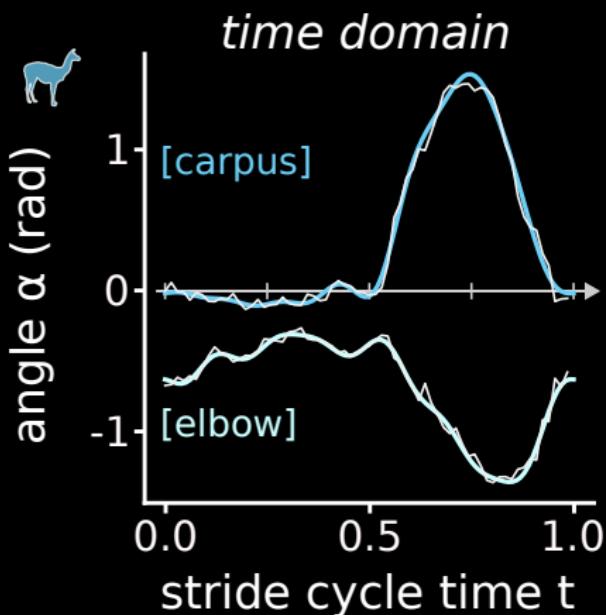
|        | Re0   | Re1    | Im1   | Re2    | Im2    | Re3   | Im3    | Re4   | Im4   | Re5    | Im5    |
|--------|-------|--------|-------|--------|--------|-------|--------|-------|-------|--------|--------|
| center | 0.247 | -0.038 | 0.320 | -0.184 | -0.071 | 0.060 | -0.071 | 0.013 | 0.029 | -0.006 | -0.005 |

superimpose set ref show leg 2d view mean elbow carpus dzg real zero quit

# Fourier Series Decomposition



# Fourier Series Decomposition



FSD:

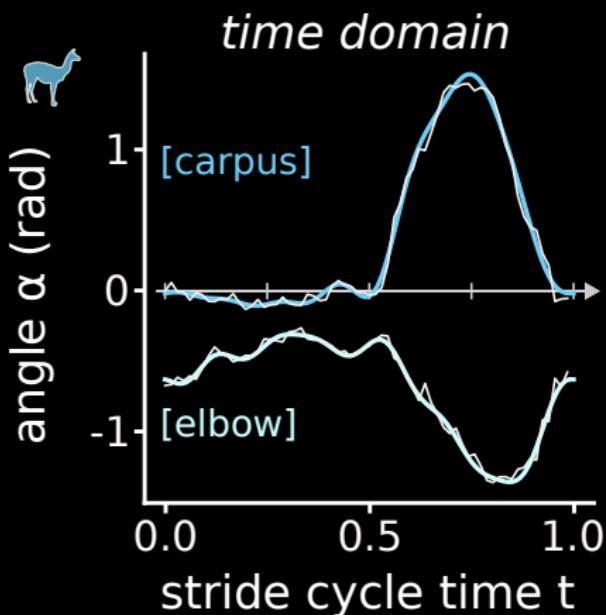
$$f(t) = \sum_{n=0}^N (2 \cdot c_n) \cdot e^{2\pi i n \frac{t}{T}}$$

$$c_n = \frac{1}{T} \sum_{t=0}^T e^{-2\pi i n \frac{t}{T}} \cdot f(t) \quad \forall n > 0$$

$$c_0 = \frac{1}{T} \sum_{t=0}^T e^{0} f(t) = \langle f(t) \rangle$$

(Osgood, 2007)

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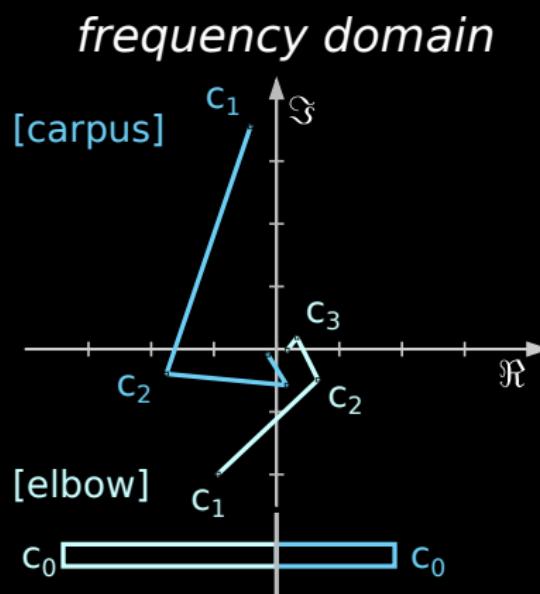
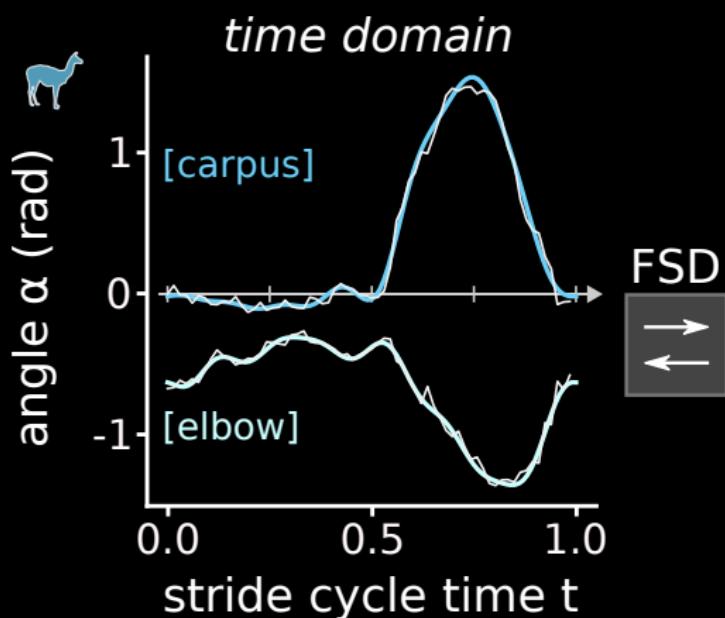
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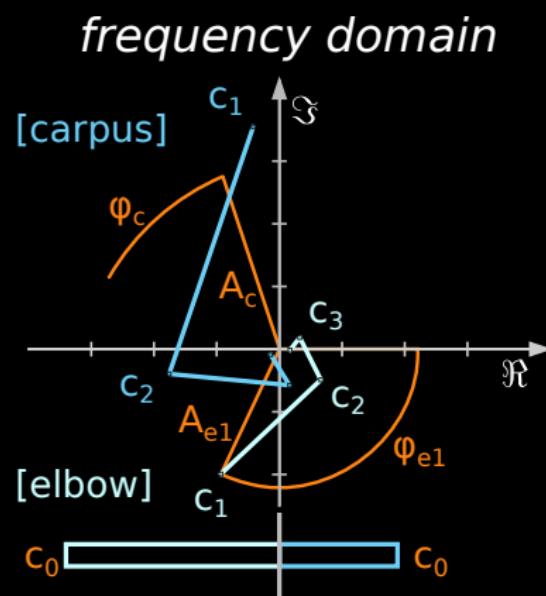
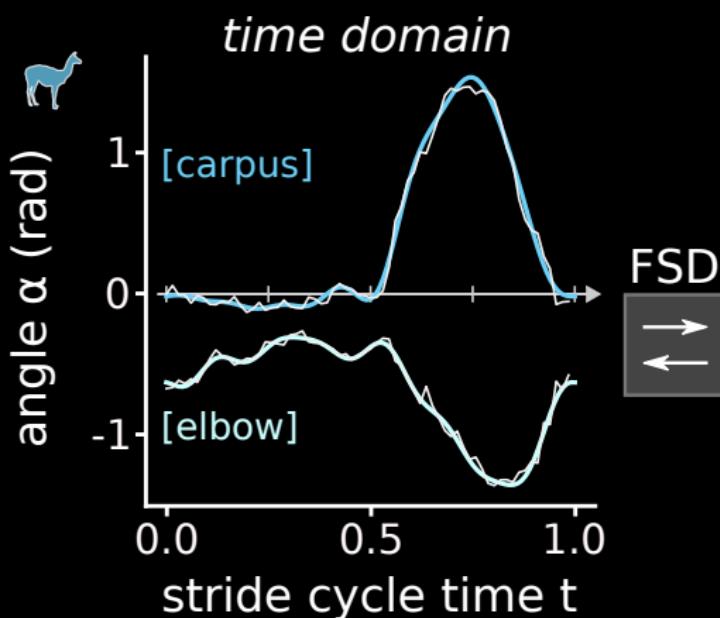
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# Fourier Series Decomposition



# Fourier Series Decomposition



# Superimposition

requires to calculate signal parameters ( $c_0$ ,  $A$ ,  $\phi$ ) from coefficients.

mean       $c_0 = \langle f(t) \rangle$

amplitude       $A = \sum_{n=1}^N \sqrt{\Re(c_n)^2 + \Im(c_n)^2} = \sum_{i=1}^N |A_i|$

phase       $\phi_n = \frac{1}{2\pi} \cdot \text{arctan2}(-\Im(c_n), \Re(c_n))$

$$\phi = \frac{\sum_{n>0}^N (\phi_n - \phi_{n-1}) \cdot \frac{A_n}{n}}{\sum_{n>0} \frac{A_n}{n}}$$

*affine components*

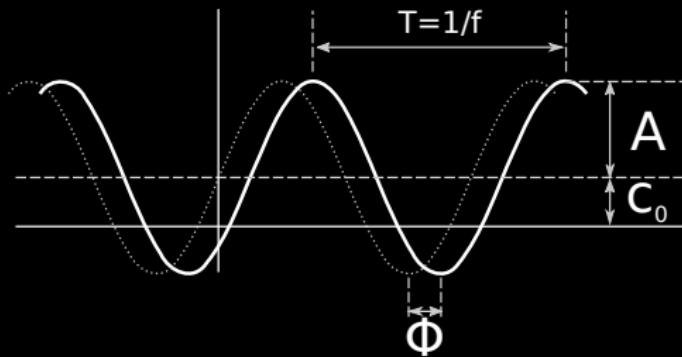
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mean

amplitude

phase



*affine components*

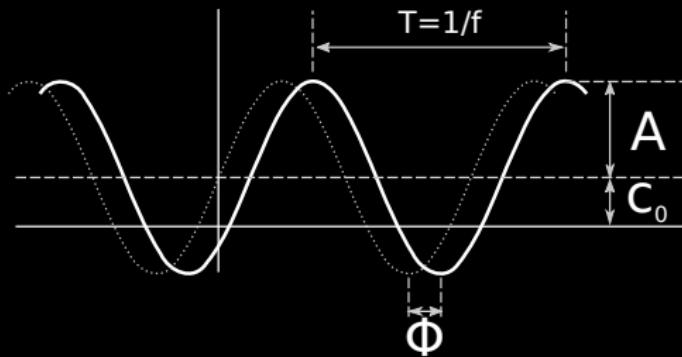
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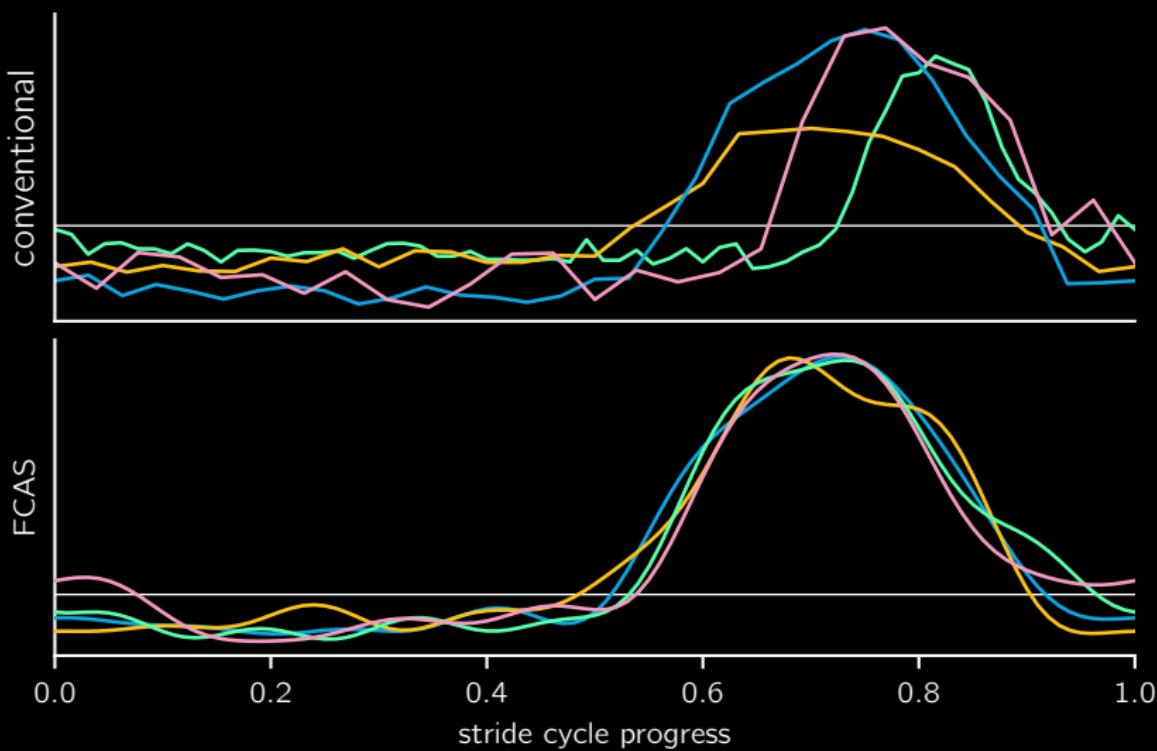
amplitude

phase

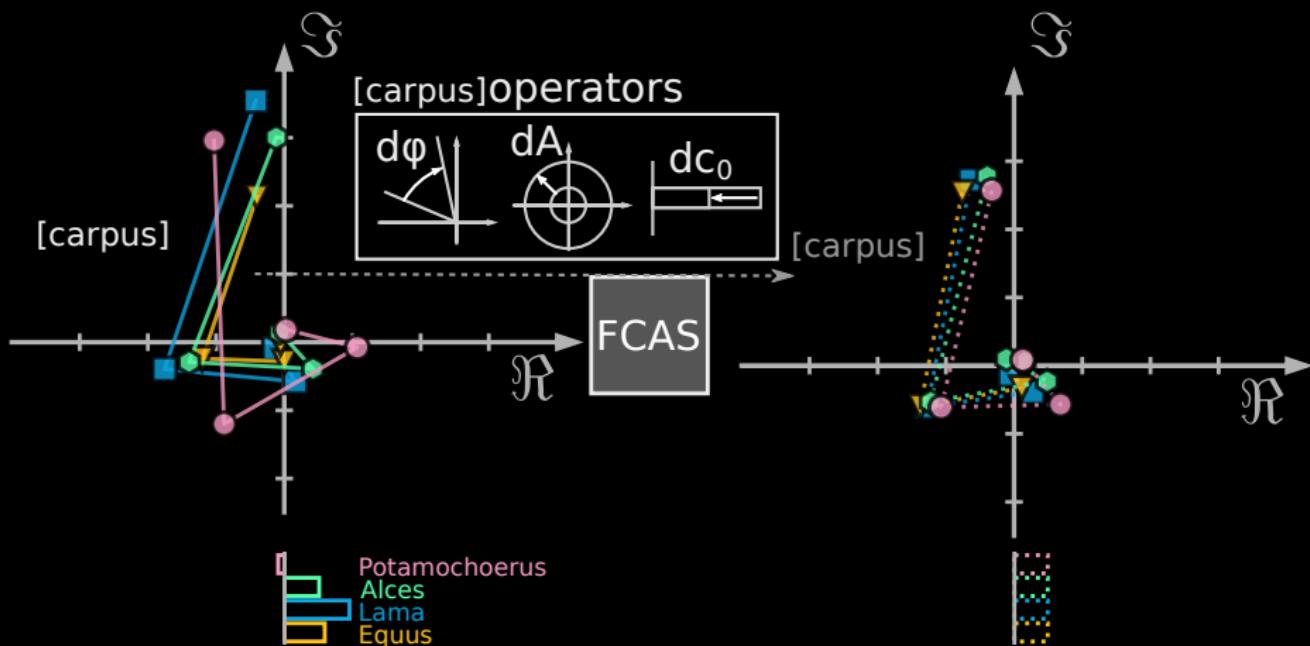


*affine components*

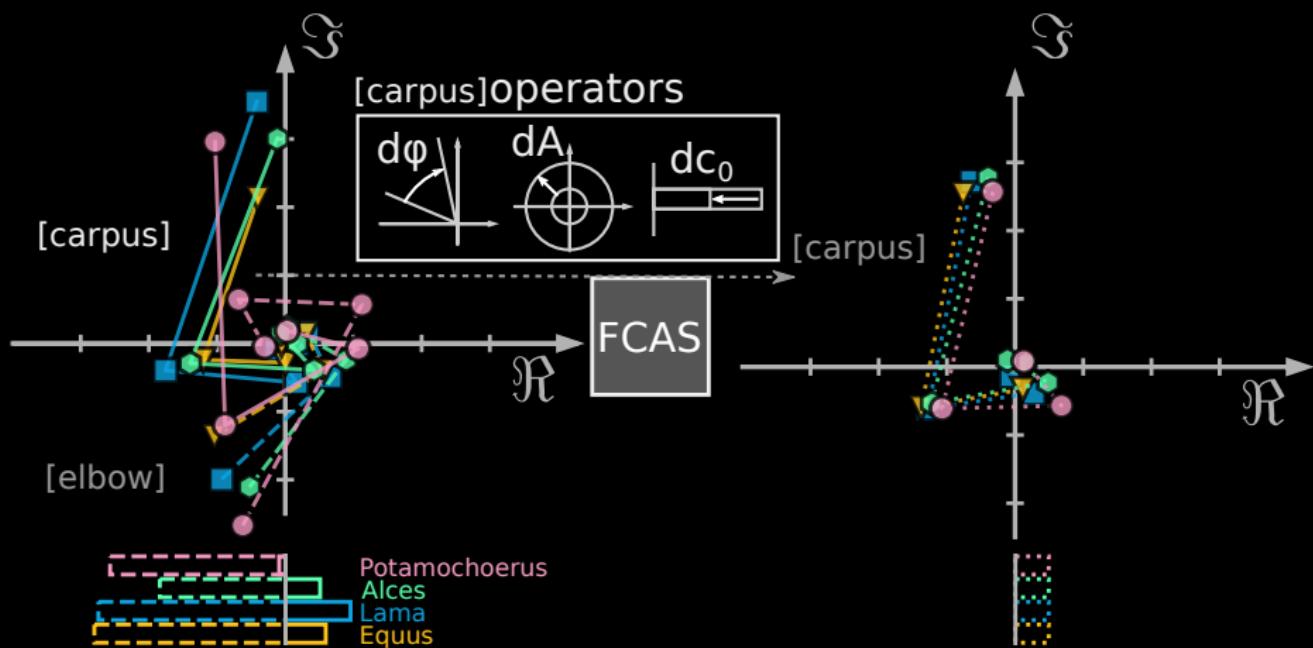
# Fourier Coefficient Affine Superimposition



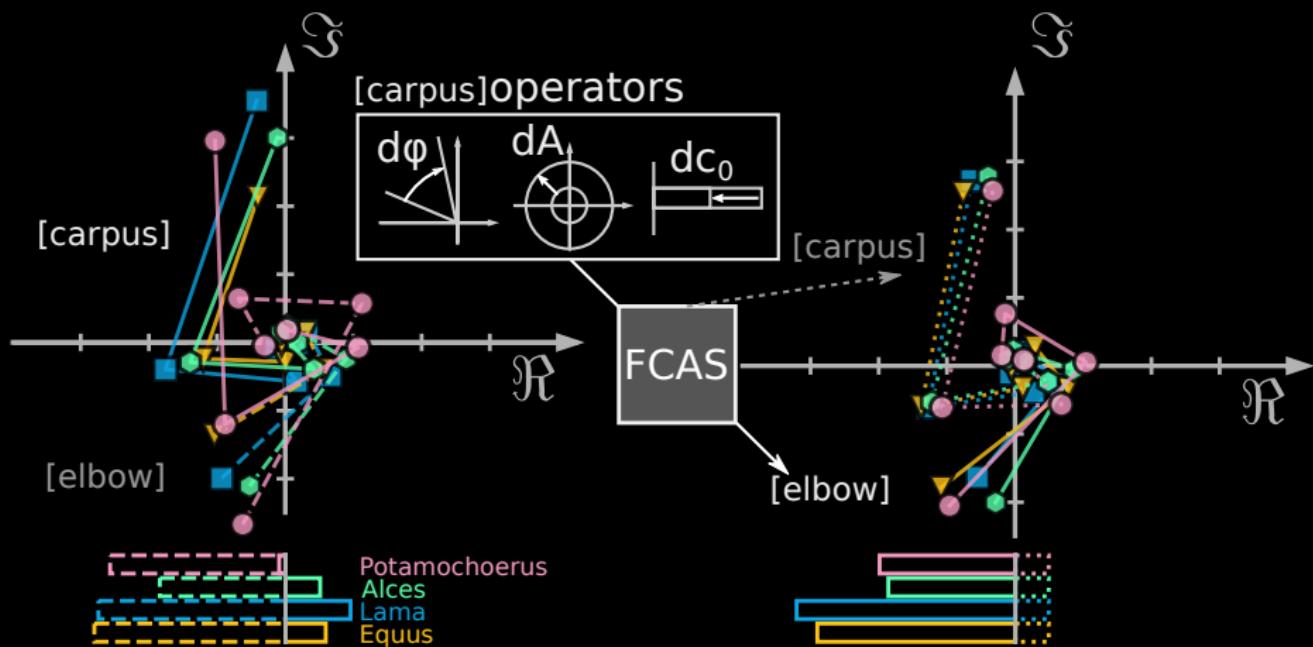
# Transfer of Operators



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# Relative Joint Profiles

- ▶ shape of elbow profiles
- ▶ relative timing
- ▶ relative amplitude
- ▶ optionally centered (posture)

*dynamic similarity*

*Inter-Joint Coordination*

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# The Shape of Locomotion

GM: geometric morphometrics  
FCAS: Fourier Coefficient Affine Superimposition

## Procedure:

|   |                       | GM              | FCAS           |
|---|-----------------------|-----------------|----------------|
| ① | acquire data          | ct/surface scan | videography    |
| ② | digitization          | landmarks       | point tracking |
| ③ | alignment             | Procrustes      | FCAS           |
| ④ | multivariate analysis | flexible        |                |



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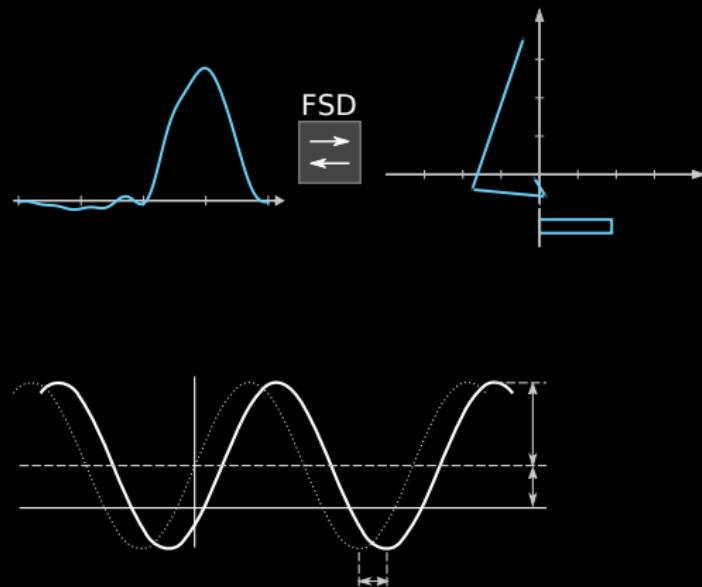
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# Summary

- ▶ Fourier Series
- ▶ affine components
- ▶ relative joint profiles
- ▶ geometric intuition



# References

- Mielke, F., Van Ginneken, C., and Aerts, P. (2019). Quantifying intralimb coordination of terrestrial ungulates with Fourier Coefficient Affine Superimposition. *Zoological Journal of the Linnean Society. Under Revision.*
- Osgood, B. (2007). The Fourier Transform and its Applications. *Stanford Engineering, course EE261.*



@UAFunMorph



[mielke-bio.info/falk/dzg](http://mielke-bio.info/falk/dzg)

Thank you!